Toward an improvement over Kerner-Klenov-Wolf three-phase cellular automaton model

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The Kerner-Klenov-Wolf (KKW) three-phase cellular automaton model has a nonrealistic velocity of the upstream front in widening synchronized flow pattern which separates synchronized flow downstream and free flow upstream. This paper presents an improved model, which is a combination of the initial KKW model and a modified Nagel-Schreckenberg (MNS) model. In the improved KKW model, a parameter p_{flag} is introduced to determine the vehicle moves according to the MNS model or the initial KKW model. The improved KKW model can not only simulate the empirical observations as the initial KKW model, but also overcome the nonrealistic velocity problem. The mechanism of the improvement is discussed.

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Traffic flow research has a quite long history (see, e.g., [1–10]). However, only recently the spatiotemporal features of congested patterns have been adequately observed and understood, especially by the group of Kerner and Rehborn [4,8–10]. As a result, earlier traffic flow models fail to explain some empirical spatiotemporal traffic pattern features [11–13].

To explain the congested traffic, Kerner [4,16], Kerner and Klenov [14,17], and Kerner *et al.* [15] introduced a three-phase traffic theory which postulates that the steady states (homogeneous and stationary states, time-independent solutions in which all vehicles move with the same constant speed) of synchronized flow cover a two-dimensional region in the flow-density plane.

In 2002, Kerner and Klenov developed a first microscopic model of the three-phase traffic theory, which can reproduce empirical spatiotemporal patterns [14]. However, this model is relatively complex. Several months later, Kerner, Klenov, and Wolf (KKW) developed a cellular automaton model of the three-phase traffic theory [15]. The KKW model uses the general rules of motion as shown in [14] and it is much simpler than the spatial continuum model.

Recently, some microscopic models based on three-phase traffic theory have been developed, which can show the congested pattern features found by Kerner. For example, Lee *et al.* proposed a different cellular automaton (CA) model which also can describe the empirical spatial-temporal pattern features of traffic [18]. Besides, Jiang and Wu [19] presented a CA model based on the comfortable driving model of Knospe *et al.* [20], which can reproduce the synchronized flow quite satisfactorily. Davis developed a microscopic model in which the upper boundary of a two-dimensional (2D) region of steady state solutions is related to a desired driver speed [21].

In this paper, we focus on the KKW model. As indicated by Kerner *et al.*, in the KKW model, the upstream front in a widening synchronized flow pattern (WSP) and general pattern (GP) which separates synchronized flow downstream and free flow upstream moves with a relatively high velocity (see Fig. 1, also Fig. 8 in [15]). This is maybe one reason why the KKW model is not used so widely as its continuous version [14,17,22–24]. This paper improves the KKW model by combining the initial KKW model with a modified Nagel-Schreckenberg model. It is shown that the improved KKW model overcomes the problem mentioned above while the qualitative description of congested patterns remains.

In Fig. 1, we show the spacetime plots of traffic flow induced by a bottleneck in the initial KKW model. Here the simulations have been performed for a one-lane road. Since we concentrate only on the evolution of the upstream front separating synchronized flow downstream and free flow upstream, the type of bottleneck is not important. Therefore, for simplicity, the bottleneck is fulfilled by a speed limit.

In the simulations, the KKW-1 model with parameter-set I (see Table III in [15]) is used. Each cell corresponds to 0.5 m and a vehicle has a length of 15 cells. One time step corresponds to 1 s. The following open boundary condition is used. At the entrance, a new vehicle with free velocity v_{free} is inserted at $x_{\text{last}}-x_{\text{in}}$ when the last vehicle is beyond x_{in} (i.e., $x_{\text{last}}>x_{\text{in}}$), where x_{last} is the position of the last vehicle. The first vehicle is moving without hindrance and it is removed when it reaches $x=L$, and the second vehicle becomes the new leading one. Here $L=40\,000$ is the system size. The speed limit is fulfilled as follows. When a vehicle is within the region $36\,000<x_n<36\,500$ and its velocity is larger than the speed limit v_{lim}, then its velocity is set to v_{lim}.

In Fig. 1(a), $v_{\text{lim}}=30$, $x_{\text{in}}=100$. One can see that the traffic flow is a widening synchronized flow pattern (WSP). In Fig. 1(b), $v_{\text{lim}}=18$, $x_{\text{in}}=100$. The traffic flow is a dissolving general pattern (DGP). In Fig. 1(c), $v_{\text{lim}}=10$, $x_{\text{in}}=100$. The traffic flow is a general pattern (GP). In the three cases, the upstream front which separates synchronized flow downstream and free flow upstream moves with a velocity approximately 50 km/h, which is unrealistically high. Moreover, the velocity as well as the induced synchronized flow is essentially independent of the value of v_{lim}. Furthermore, with the increase of x_{in} (this corresponds to a smaller flow rate), the velocity becomes even larger [Fig. 1(d)].

Next we improve the KKW model by combining the initial KKW model with a modified Nagel-Schreckenberg (MNS) model. To this end, a parameter p_{flag} is introduced. When $p_{flag}=1$, the vehicle moves according to the MNS model; when $p_{flag}=0$, the vehicle moves according to the initial KKW model. The value of p_{flag} is decided as follows:

(i) If $\lfloor (x_{n+1}-x_n-l)/T \rfloor < v_n$, then $p_{flag}=0$

(ii) Else if $v_n=v_{\text{free}}$, then $p_{flag}=1$

(iii) Otherwise p_{flag} remains unchanged.

Here $\lfloor x \rfloor$ denotes the maximum interger not larger than x, T is

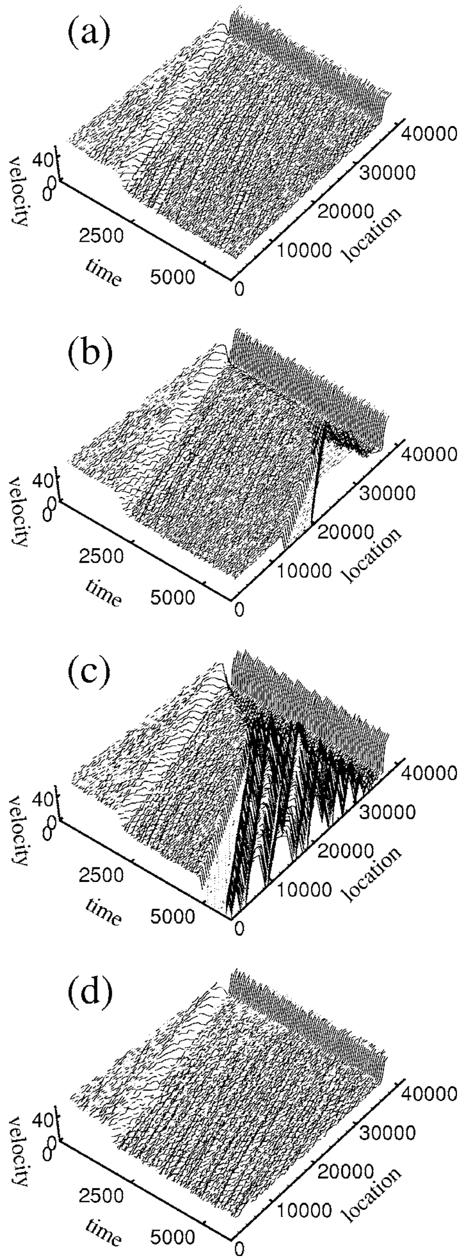

FIG. 1. Traffic flow induced by a speed limit in the initial KKW model. (a) $v_{\mathrm{lim}}=30$, $x_{\mathrm{in}}=100$; (b) $v_{\mathrm{lim}}=18$, $x_{\mathrm{in}}=100$; (c) $v_{\mathrm{lim}}=10$, $x_{\mathrm{in}}=100$; (d) $v_{\mathrm{lim}}=30$, $x_{\mathrm{in}}=110$. $x_{\mathrm{in}}=100$ corresponds to flow rate 2137 vehicles/h, $x_{\mathrm{in}}=110$ corresponds to flow rate 1942 vehicles/h.

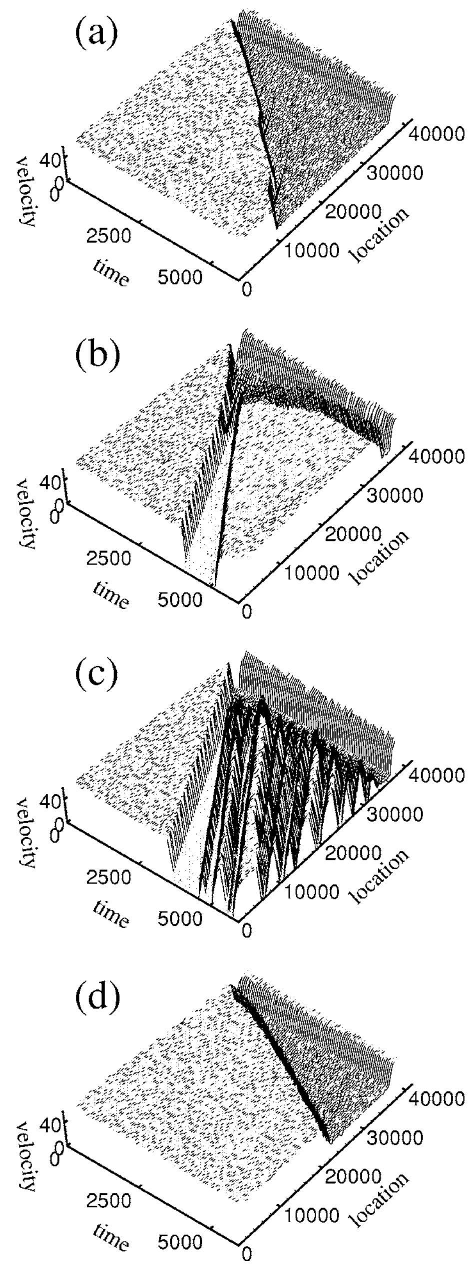

FIG. 2. Traffic flow induced by a speed limit in the improved KKW model. (a) $v_{\mathrm{lim}}=30$, $x_{\mathrm{in}}=86$; (b) $v_{\mathrm{lim}}=20$, $x_{\mathrm{in}}=86$; (c) $v_{\mathrm{lim}}=10$, $x_{\mathrm{in}}=86$; (d) $v_{\mathrm{lim}}=30$, $x_{\mathrm{in}}=90$. $x_{\mathrm{in}}=86$ corresponds to flow rate 2454 vehicles/h, $x_{\mathrm{in}}=90$ corresponds to flow rate 2388 vehicles/h.

the preferred time headway of the driver, x_n and v_n are the position and velocity of vehicle n (here vehicle $n+1$ precedes vehicle n), and l is the length of the vehicle. We believe the switch between the two models is consistent with a real driver behavior. When the driver is in congested flow (i.e., $p_{flag}=0$), the driver may not accelerate even if its time gap allows acceleration (i.e., $\lfloor (x_{n+1}-x_n-l)/T \rfloor \geqslant v_n$) because he may have to decelerate again when he catches up with his preceding vehicle. Therefore he accelerates only when the preceding vehicle is faster (i.e., moves according to the initial KKW model). When the driver leaves the congested flow, his velocity gradually increases to maximum velocity (i.e., $v_n=v_{\mathrm{free}}$) and accordingly his behavior switches to follow MNS model (i.e., p_{flag} changes into 1).

In the MNS model, cars are updated by four consecutive rules as followings: (1) Acceleration: $v_n \rightarrow \min(v_n+1, v_{\mathrm{free}})$; (2) slowing down: $v_n \rightarrow \min(\lfloor (x_{n+1}-x_n-l)/T \rfloor, v_n)$; (3) randomization: if $v_n > 0$, then $v_n \rightarrow v_n - 1$ with probability p_1; and (4) motion: the position of a car is shifted by its speed v_n. These four update rules are applied in parallel to all cars. It is clear that the MNS model is different from the NS model in step (2). In the NS model, the preferred time headway T is set to 1 s. However, in the real world T is normally

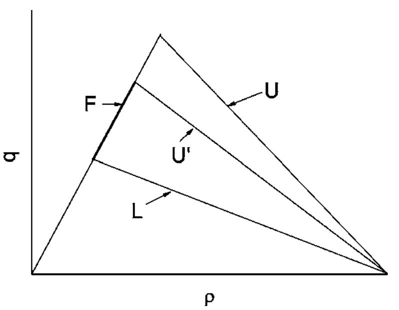

FIG. 3. The region of steady state solutions of the improved KKW model and the initial KKW model.

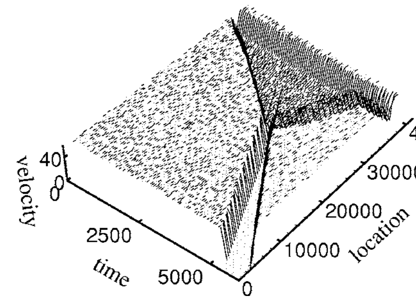

FIG. 4. Traffic flow induced by a speed limit in the improved KKW model. $v_{\rm lim}=30$, $x_{\rm in}=86$.

longer than 1 s. In this paper, T is set to 1.2 s.[1]

In Fig. 2, we show the simulation results of traffic flow induced by a speed limit in the improved KKW model. Here the randomization parameter in the MNS model is set to $p_1=0.3$. Similarly as in the initial KKW model, each cell corresponds to 0.5 m and a vehicle has a length of 15 cells.

In Fig. 2(a), $v_{\rm lim}=30$, $x_{\rm in}=86$. One can see that the traffic flow is a WSP. In Fig. 2(b), $v_{\rm lim}=20$, $x_{\rm in}=86$. The traffic flow is a DGP. In Fig. 2(c), $v_{\rm lim}=10$, $x_{\rm in}=86$. The traffic flow is a GP.

In Fig. 2(a), the upstream front which separates synchronized flow downstream and free flow upstream moves with a realistic velocity (approximately 8.5 km/h). In Figs. 2(b) and 2(c), there is no synchronized flow between the free flow and the first wide moving jam. These are consistent with the empirical observations. Furthermore, with the increase of $x_{\rm in}$, the velocity of the upstream front becomes smaller [approximately 5 km/h, Fig. 2(d)].

The mechanism of the improvement is the region of the steady solutions of the improved KKW model changes comparing with the initial KKW model. In Fig. 3, we show the region of steady state solutions of the improved KKW model and the initial KKW model. In the initial KKW model, the 2D region of steady state solutions is limited by three boundaries, the upper line U (which corresponds to safe speed), the lower curve L (which corresponds to synchronized flow distance), and the left line F (which corresponds to free flow). In the improved KKW model, the 2D region separates into two regions by the curve U', which corresponds to the preferred velocity of the vehicle $\lfloor (x_{n+1}-x_n-l)/T \rfloor$. When $p_{flag}=1$, the region of the steady solutions reduces to a line represented by the thick line. When $p_{flag}=0$, the steady solutions correspond to the 2D region limited by three boundaries U, L and F, but excluding the thick line.

In traffic flow there exist metastable regions and the metastable states partially overlap one another (see Sec. 18.5 in Ref. [4]). In the overlapping metastable states, either free flow or one of the synchronized flow patterns or else one of the GPs (or DGPs) can occur. The improved KKW model can describe the metastability of traffic flow. For example, Fig. 4 shows a different traffic flow situation from Fig. 2(a). However, both situations start from the same initial conditions and boundary conditions, only different seeds in the Fortran program are used. This property is consistent with the empirical observations and the three-phase theory of Kerner [4] and Kerner and Rehborn [8].

Finally, we would like to talk about the limit flow rate $q_{\rm lim}^{(\rm pinch)}$ reached in the pinch region in the on-ramp system [17]. The existence of $q_{\rm lim}^{(\rm pinch)}$ is due to the on-ramp traffic itself becoming congested when the flow on the on ramp exceeds a limit $q_{\rm on}^{(\rm lim)}$. Therefore, when $q_{\rm on}>q_{\rm on}^{(\rm lim)}$, increasing $q_{\rm on}$ will not increase the flow rate from the on ramp to the main road, instead it increases the queueing speed on the on ramp. In contrast, in the case of speed limit bottleneck, there is no existence of limit flow rate $q_{\rm lim}^{(\rm pinch)}$, the flow rate in the pinch region can be very small (see, e.g., Fig. 5).

In summary, we have presented an improved KKW model, which is a combination of the initial KKW model and the MNS model. The improved KKW model can be used to simulate the empirical observations as the initial KKW model. Moreover, it overcomes the existing problem in the initial KKW model, namely, the upstream front which separates synchronized flow downstream and free flow upstream moves with a relatively high velocity. The mechanism of the improvement is the region of the steady solutions of the improved KKW model changes when comparing with the initial KKW model.

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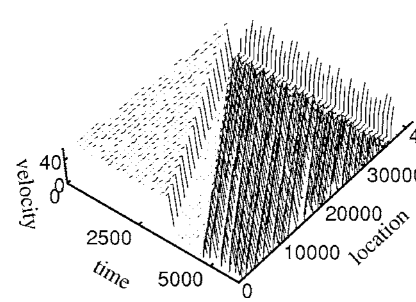

FIG. 5. Traffic flow induced by a speed limit in the improved KKW model. $v_{\rm lim}=5$, $x_{\rm in}=86$. The flow rate in the pinch region is 833 vehicles/h, which is smaller than $q_{\rm lim}^{(\rm pinch)}=1153$ vehicles/h in the case of the on-ramp system.

[1]There is no significant difference between the choice of $T=1$ s and $T=1.2$ s. The problem when choosing $T=1$ s is the maximum flow rate that can be reached in the free flow is about 2850 vehicles/h, which is higher than the realistic value (about 2400 vehicles/h).

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